THE "PAYI" T1C-T0C LESSONS

LESSON 2

INTRODUCING THE BINARY SYSTEM

Diego Pareja-Heredia. *Universidad del Quindío. Armenia. Colombia.*
E-mail: depehache@yahoo.es

Sharing Payi's experiences. Children's joy is reflected in this picture, when the little clown T1C-T0C visits the working table, where the kids are practicing T1C-T0C movements.

Introduction. In Lesson 1 we saw Payi making the intuitive basis to build up, in a natural and simple way, the binary number system, using elementary entities: T**1**C and T**0**C associated to the symbols **1** and **0**, respectively. We have to point out that symbols are not the numbers by themselves, but numerals, that represent them through the intelligible language we use in our either oral or written communication. Numbers are abstract entities, no easy to define, and which humans have been using since early stages of their evolution to denote the size of magnitudes, and because of social pressures and also because the need for philosophical and scientific speculations.

Mathematics is an integral part of human culture, no matter what degree of development it has. Mathematics has shown up, what Eugene Wigner once called "the unreasonable effectiveness" as a tool to solve a wide range of problems. Nevertheless, we have to recognize that when we mention mathematics as a tool, we are not saying that math is just only that; a great part of it is more a kind of art, since in these terrains we do not see immediate applications but so much beauty.

A definition for mathematics is not easy to get. Richard Courant and H. E. Robbins wrote a whole book trying to answer the question *What is Mathematics*? in the cultural context around the middle of twenty century. Nowadays we have to look for another answer to the same question inside a new context originated in the vast empire of the new developments of mathematics. I don't favor definitions; however, let me give the following one, which comes from one of my known references[1]: *mathematics is the science of order, patterns, structure, and logical relationships.*

In this part of these lessons, we'll refer to natural numbers. Inside T**1**C-T**0**C's name, are included the numbers **1** and **0**, represented on it as complementary entities, as much in form as in color. The duality *forward- backward*, *right-left*, represented in **1**, **0**, as seen in the first lesson, let us construct one part of the binary tree. To the left part of the tree we can arrive opening another branch up the number **0**. From this binary tree we can derive all the real numbers, as expansions of base **2**.

One aspect that we want to point out is the resource of kid's right hand (or left hand) as a learning tool to get a better comprehension of the procedure to carry out the construction of the binary system to represent whole numbers. Through years at classroom, our hands have been almost exclusively used as a writing device. The aim of our method is emphasize in the possibility of using our two hands to create new patterns as graphs, in this case a binary tree, which evolves from a dynamics related with movements along the plane.

Trough this learning process, beginning in this lesson; the fingers, especially index and middle fingers, play an important role. Those fingers show us whether we move forward, backward, right or left. The first lesson introduces the convention that T1C means forward or right and T0C means backwards or left and these meanings are transferred to **1** and **0**.

The Second Dialog with Payi. In the following dialog "Payi" T1C-T0C is going to teach the way how to create, beginning with **0** and **1,** the set of numerals used in **NUMB3R PL4N3T, -** the fantasy world – where he lives. As in the first lesson the characters are the little clown Payi and Tea, the teacher, who makes the questions and animates the presentation. The scene is now a tessellated floor, where Payi is standing up. This floor starts with a long tile followed by two long tiles, and after this, more bands with four, eight, sixteen, etc., tiles each. The rows are named with music signs showing the likeness between this tessellation and the time intervals in music.

Payi is standing in the first row, where numerals **0** and **1** are (See NUMB3R PL4N3T TILING at the end). When Payi makes the first step (T**1**C), he finds two tiles. He goes either to left, if he chooses T**0**C, or right if he does T**1**C. If he goes further he finds out always two new options to continue. On the table kids will find a copy of Payi's floor, where they can reply Payi's movements.

---

[1] DEVLIN, K. *The Math Gene. How Mathematical Thinking Evolved and what Numbers are like Gossip*. Basic Books. 2000. Pag. 74.

Tea – Hello Payi, we learned in the first lesson that T**1**C means a movement either forward or right and also that, T**0**C means one step backwards or a step to left. Besides, we accepted to use **1** and **0** instead of T**1**C and T**0**C respectively. We expect you teach us how you represent whole numbers using just symbols **0** and **1**.

T1C-T0C – That's right my dear Tea. As I told you, in **NUMB3R PL4N3T** we learned mathematics playing, so the first thing we need is a playground. We need a lined surface.

Tea – A lined surface as the floor of our houses or as the school floor?

T1C-T0C – It could be for you. I am so short that I need some kind of floor as that one you have on your table.

Tea – OK. But I imagine that no all tiles are equal. Are they?

T1C-T0C – It could be, although with some rules. As you know; all games have some rules, and these have to be held in order to play the game correctly. We try to get very simple rules por our games.

Tea – More simple than squared tiles on my house floor?

T1C-T0C – It is something simpler than that. We start with a long tile where I am standing. In this first tile are **0** and **1** **(**T0C and T1C**)**. When I begin with the first step forward I'll find two tiles one at left and the other one at right.

Tea – Oh my god, it's a very strange floor.

T1C-T0C – Not really, as you'll see later, it is not so rare. For each step forward, we're going to find twice the tiles we have before this step.

Tea – That means we begin with one and go doubling the number of tiles as we go a step forward, something like 1, 2, 4, 8, etc.

T1C-T0C – It's so as simple, but remember my dear Tea that we do not know yet how to write numbers in the binary system. Only we have at hand **0** and **1**.

Tea – That's right Payi, I know that in **NUMB3R PL4NET** you do not use the same numerals as in here. You told us you do not use ten fingers but just your own two arms: T1C and T0C.

T1C-T0C – In my world we learn to double by repetition. It is so easy. We start with two options, if we need more than that, we just give to both of them, two more options each, and continue to infinity.

Tea – Did you say to *infinity*?

T1C-T0C –Yes, with the meaning of a process of doubling indefinitely, a process that never ends.

Tea – Oh I see, to repeat the same thing over and over again.

T1C-T0C – That's right. When we start with the first step we find out two new options either T1C or T0C, according we chose right or left.

Tea – I understand, when we arrive to the second row we make T1C, so we have **1**. If we go one step right we say T1C again and we have **11** for the right tile at the second row. If we go instead left we arrive to **10** for the left tile at the second row.

T1C-T0C – That's Ok, Tea. Let me point out that we have to say "One zero" not "Ten", and "One one" and not "Eleven". The ten and eleven are numeral words that correspond to the decimal system and not to binary system we are introducing.

Tea – OK, we have **0**, **1**, **10**, **11**, in that order. Is it right?

T1C-T0C – Oh, yes. On the other hand, when we advance with another step either from **10** or from **11**, we have two choices to follow up, according we take either T1C or T0C.

Tea – Till here, everything is Ok. Let's see how our kids can handle the process for building up the binary tree. We are going to follow Payi's movements: one step forward starting in **1** followed of another step left to complete T1CT0C and we are at **10**; again at **1**, one step forward and then right to complete T1CT1C and we are at **11**. Come on kids let's repeat our game starting with T0C, then: T1C, T1CT0C, T1CT1C, in short, we have constructed the sequence: **0**, **1**, **10**, and **11**.

T1C-T0C – That's good kids. In **NUMB3R PL4NET** we usually learn step by step with T**1**C and T**0**C and after some practice we left the letters T and C and follow on, just with **1** and **0.**

Tea – Why did you omit letters T and C?

T1C-T0C – As I told you in the first lesson, in mi name appears symbols for numbers **1** and **0**. So, when I show **0**, I mean T0C and when I show **1**, I am meaning T1C. Of course, it is easier to write **1** and **0** than T1C and T0C.

Tea – Very well, Payi, from now on we'll use **1** for a forward or right step and **0** for a backwards or a left step.

T1C-T0C – We always try to simplify our language. It is easier to handle short symbols than longer ones.

Tea – Ok Payi, we have arrived to **11**, what does follow from here?

T1C-T0C – When we are standing at **10**, we can go one step further either with **0**, to **100**, or with **1** to **101**. If we are at **11**, again there are two choices, either to **110** or to **111**.

Tea – That means we have found four new numbers **100**, **101**, **110** y **111**.

T1C-T0C – With these new numbers our number sequence goes like this: **0**, **1**, **10, 11**, **100**, **101**, **110** and **111**. From the last four numbers we can find eight new ones, using the two choices **0** and **1** after these numbers, in this way: **1000**, **1001, 1010**, **1011**, **1100**, **1101**, **1110** and **1111**.

Tea – Ok kids; let us practice with **0** and **1** using as a play ground, the lined sheet you have on the table at front of you. Try to fill out next rows after the last numbers we have written down before.

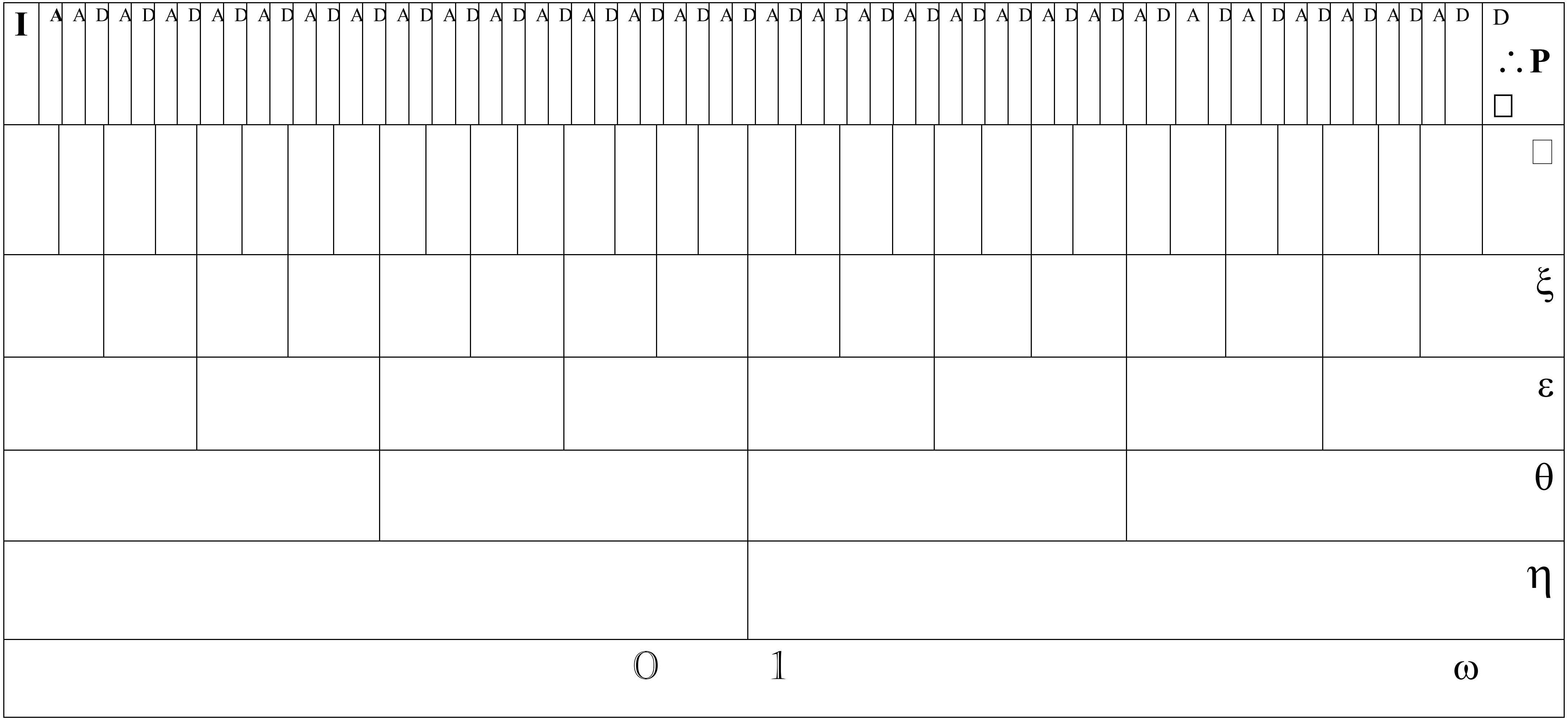
T1C-T0C – With each new number always grow up two additional ones, like two sprouting branches in each number. This whole process looks like a tree and that is the reason why the resulting graph is called a binary tree.

Tea – Tree diagrams are important in mathematics. Our new technology and advanced mathematics use these kinds of trees. At the back of Payi's tessellation sheet you'll find a binary tree. Ok Payi, your binary game was very interesting. Thank you for this new lesson.

T1C-T0C – It was nice to visit you and I hope you learn addition and multiplication with binary digits by yourselves. It is so easy that machines can also learn it. Bye and good luck to all of you.

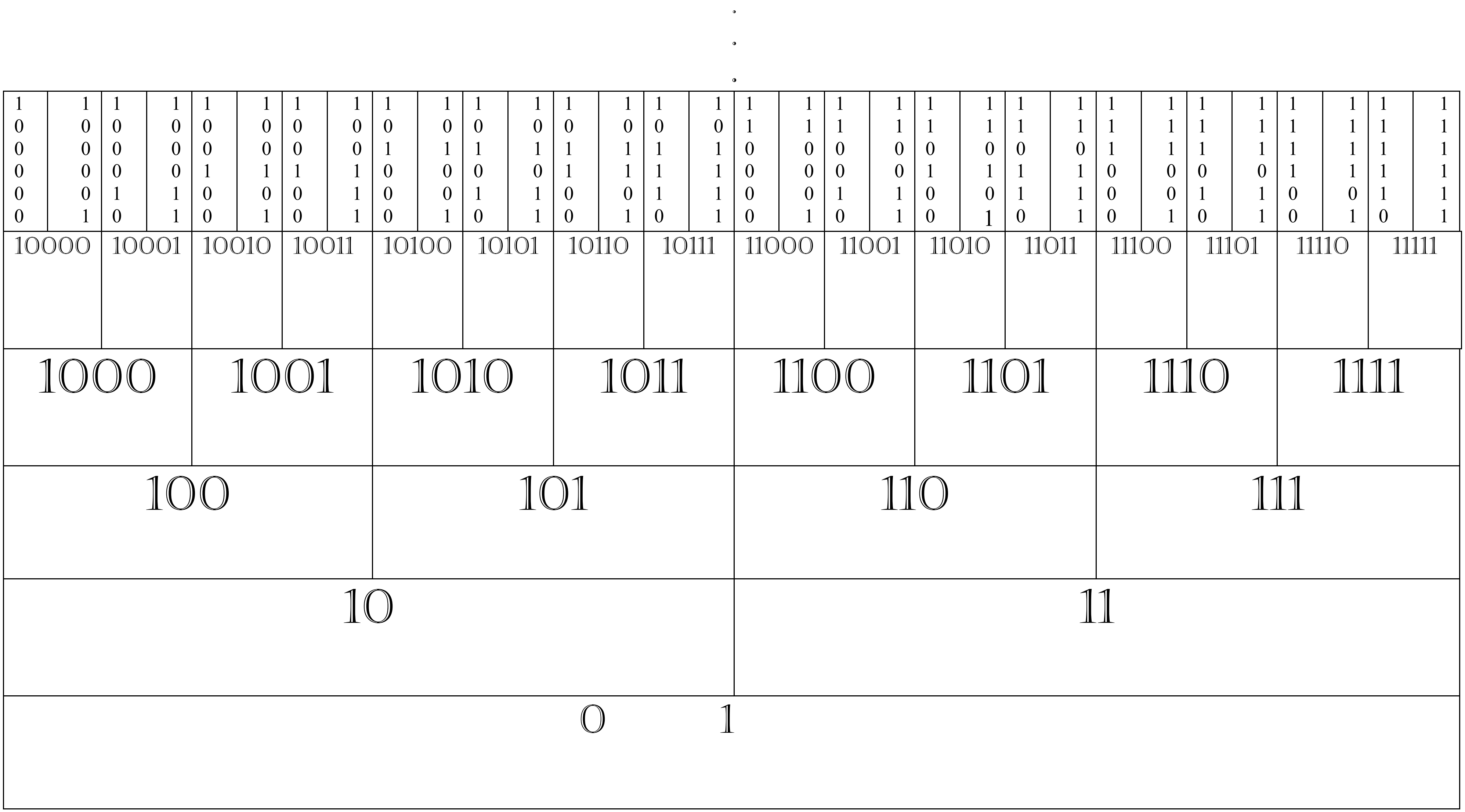
The Payi's kisses. The little clown Payi was sharing his affection with the pretty girls attending his performance at Armenia, Colombia on May 28, 2008.

NUMB3R PL4N3T TILING
AND "T1C-T0C HELL-PARADISE" GAME)



Number PL4N3T Tiling and T1C-T0C Hell-Paradise Game. Payi uses this tiling to construct the whole numbers. Starting with **0** with a right step he reaches **1**. From here on, with a step forward he can go to either to **10** or **11**, according he chooses a left or a right step and from these numbers up, two new numbers appear: the first one ending with zero and the other one ending with one, both of them carrying on as their first digits the numbers where they started. The **1**-binary tree is shown in next figure. The Hell-Paradise game is played with a coin marked with a **0** and **1**. Each throw of the coin gives a step forward, with left or right if zero or one is shown on the coin. The letters **A, D, P**, on the superior row mean *Angel, Demon*, respectively. Wins the game the kid who arrives to either **A** or **P**. Players have to write down the path they follow until the game is over. Musical signs at right are associated to the lengths of sound intervals which are similar to the relationship inside the number tree.

BINARY TREE FOR 1



A Binary Tree for 1. Joining numbers from **1** up, so that each number is connected with two adjacent in the upside row, gives a graph called a tree. In this case we get the binary tree for **1**. A similar tree can be raised for **0**. The binary tree let us construct real numbers as base two expansions.